

**Rubidium resonant squeezed light from a diode-pumped optical-parametric oscillator**

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We demonstrate a diode-laser-pumped system for generation of quadrature squeezing and polarization squeezing. Due to their excess phase noise, diode lasers are challenging to use in phase-sensitive quantum optics experiments such as quadrature squeezing. The system we present overcomes the phase noise of the diode laser through a combination of active stabilization and appropriate delays in the local oscillator beam. The generated light is resonant to the rubidium  $D1$  transition at 795 nm and thus can be readily used for quantum memory experiments.

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**I. INTRODUCTION**

Interaction of quantum states of light is of interest both for quantum communications, for improved sensitivity in measurements limited by quantum noise, and for understanding light-matter interactions at the most fundamental level. Our interest is in quadrature squeezing and polarization squeezing, which are phase-dependent quantum features. A proven technique for generation of squeezing is phase-sensitive amplification in a subthreshold optical-parametric oscillator (OPO) [1]. This technique has benefited in recent years from advances in nonlinear materials, low-loss coatings, and low-noise detectors. Dramatic improvements in degree of squeezing [2,3] and squeezing in new frequency bands [4,5] have been demonstrated. For strong interaction with atoms the squeezed light needs to be atom resonant, which limits the choice of lasers, nonlinear crystals, and detectors. Several experiments have demonstrated squeezing at the rubidium resonance [6–9]. These experiments use distinct methods: squeezing in a waveguide [6] and down-conversion in an OPO [7–9]. In the latter case, the squeezing at the rubidium  $D1$  line using the nonlinear interaction in a subthreshold OPO was achieved by using a Ti:sapphire laser and periodically poled potassium titanium oxide phosphate (PPKTP) as nonlinear medium [7–9]. The noise suppression of this method was shown to be more than  $-5$  dB [8]. To our knowledge the only experiment that generated squeezing in an OPO pumped by a diode laser system worked at 1080 nm [10], far from any useful atomic resonance and produced relative intensity squeezing, a phase-independent property.

Compared to other laser systems, diode lasers are easy to operate, compact, and inexpensive. They also allow for tunable operation in a variety of wavelength ranges, and many important atomic transitions can be addressed with diode lasers. For these reasons, diode-laser-based squeezing would significantly extend the range of possible squeezing experiments. It has long been suspected that the excess phase noise of the diode laser, which results in a relatively large linewidth, would be an obstacle for production of phase-sensitive quantum states such as quadrature squeezing. The spectral distribution of diode laser phase noise over different frequencies was investigated in [11]. There, it was shown that the main contribution in the noise comes from the low-frequency part of the spectrum, as expected for a process of

phase diffusion. This suggests that the laser output can be treated as quasistationary, with the laser frequency drifting slowly (on the time scale of propagation and cavity relaxation) within the laser linewidth.

Here we show that cavity stabilization of the diode laser frequency, in combination with appropriate delays for the local oscillator beam, allows squeezing to be observed with diode-laser-based systems. Here we demonstrate a technique to eliminate the effects of laser phase noise on quadrature squeezing, and generate squeezing at 795 nm with a diode-based system. The technique uses cavity stabilization of the laser frequency, in combination with a carefully chosen delay of the local oscillator beam. We derive the observable squeezing produced by the parametric oscillator, including the effects of quasistatic frequency fluctuations, and show that these can be eliminated by proper choice of local oscillator delay. We then measure the observed squeezing as a function of delay and find good agreement with theory. With the proper delay, the observed squeezing reaches the level expected from measured characteristics of the OPO and detection system, indicating that the effects of phase noise have been effectively canceled. The technique could also be applied to cancel phase noise in systems not based on diode lasers.

We present our experimental system, a PPKTP-based subthreshold OPO pumped by a frequency-doubled diode laser, and expected and observed squeezing performance. We then consider the effect of the frequency fluctuations on the observable squeezing in the regime of the quasistationary fluctuations, an analysis which indicates that the system can be made immune to random frequency drifts for appropriate local oscillator delay. Finally, we present measurements of squeezing versus delay in agreement with the theory. The experimental squeezing apparatus we use, including laser, doubling system, and stabilization, use standard techniques and could be applied to a variety of other wavelengths.

**II. EXPERIMENTAL SETUP**

The schematic of the experiment is shown in Fig. 1. Our laser system (Toptica TA-SHG) consists of a grating stabilized 795 nm diode laser which is amplified by an optical tapered amplifier and injected into a frequency doubler with lithium triborate crystal as nonlinear medium. A 20 MHz

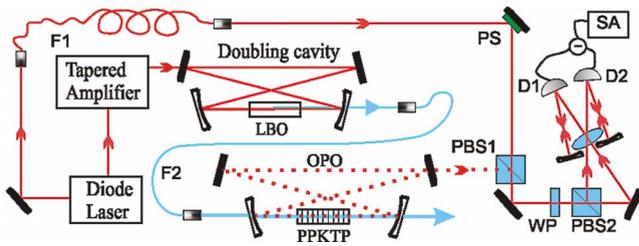


FIG. 1. (Color online) Experimental apparatus. Light from the diode laser is amplified in the tapered amplifier and fed into the doubling cavity. The blue output light is mode matched into fiber  $F2$  and fed into the OPO cavity. Both doubling and OPO cavity are only resonant to the red light. The length of the local oscillator beam path can be changed by fiber  $F1$ . The modes of squeezed vacuum and local oscillator are then overlapped on a beamsplitter (PBS1) where power balancing is performed by a wave plate (WP) and a beamsplitter (PBS2). Light is collected onto diodes  $D1$  and  $D2$  of the balanced detector. The obtained electrical signal is recorded using a spectrum analyzer (SA).

modulation is applied to the laser current, resulting in frequency-modulation sidebands of 5%. The reflection from the cavity is demodulated to provide an error signal [the Pound-Drever-Hall (PDH) technique]. The laser and cavity are locked in frequency by a proportional-integral-derivative (PID) circuit acting on the cavity piezo and a fast proportional component acting on the current of the diode. At the same time, the absolute laser frequency is stabilized by frequency-modulation (FM) spectroscopy of a saturated-absorption signal, fed back by digital PID to the piezoelectric transducer of the laser grating. For the experiments described here, the laser was locked to the  $F=2 \rightarrow F'=1$  transition of  $^{87}\text{Rb}$ . Residual fluctuations of the FM spectroscopy signal indicate that the fast cavity lock reduces the linewidth to 400 kHz full width at half-maximum (FWHM).

The generated 397 nm light is passed through a single-mode fiber for spatial filtering and pumps the subthreshold degenerate optical-parametric oscillator. The power after the spatial filtering is 45 mW which fed into OPO cavity results in parametric gain of 3. The nonlinear material used in the OPO is a 10-mm-long PPKTP crystal, temperature tuned for the maximum second-harmonic generation efficiency. The OPO cavity is a 64-cm-long bow-tie configuration resonator which consists of two spherical mirrors ( $R=10$  cm) and two flat mirrors. The distance between the spherical mirrors is 11.6 cm yielding to the beam waist in the crystal of  $42 \mu\text{m}$ . The output coupling mirror of the OPO has a transmission of 7.8%, and the measured intracavity losses are 0.55%. The measured cavity linewidth is  $\delta\nu=8$  MHz (FWHM) and the output coupling efficiency  $\eta=0.93$ . The free spectral range of the OPO cavity is 504 MHz. The cavity is locked using the Pound-Drever-Hall technique performed on the transmission signal of a counterpropagating beam fed into the cavity through the high reflecting flat mirror. The error signal is digitized and fed into a PID circuit synthesized within a National Instruments field-programmable gate array (FPGA) board type NI 7833R. It controls the OPO cavity length by moving the position of one cavity mirror with a piezoelectric transducer.

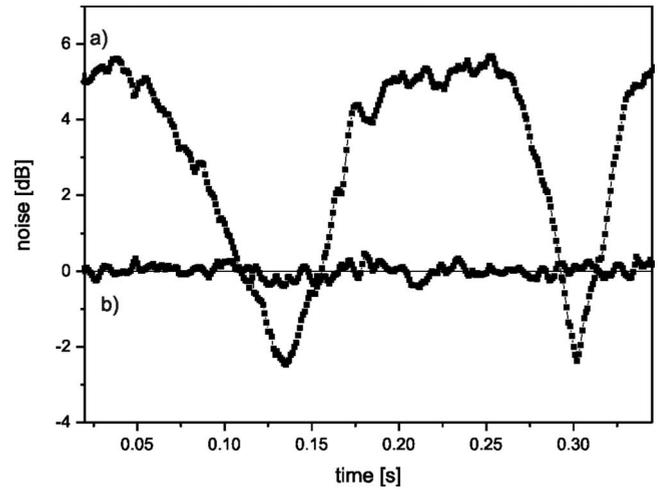


FIG. 2. Squeezed vacuum generation: (a) Squeezing trace when scanning the phase of the local oscillator, (b) shot noise level. Electronic noise is subtracted. Spectrum analyzer at zero span, resolution bandwidth is 30 kHz, video bandwidth is 30 Hz.

A local oscillator beam is derived from the diode laser by passing through single-mode fibers whose combined lengths can be chosen to give a desired group delay. The vertically polarized OPO output is overlapped with  $400 \mu\text{W}$  of this horizontally polarized beam on polarizing beamsplitter (PBS1). Optimized overlap results in a measured homodyne efficiency of  $\eta_{\text{hom}}=0.98$ . Local oscillator and squeezed vacuum beams are mixed and balanced in power on a second polarizing beamsplitter (PBS2) and detected with a ThorLabs (PDB150) switchable-gain balanced detector. The quantum efficiency of this detector at detection wavelength of 795 nm is 88% by manufacturer specifications. Losses are mainly caused by the reflection of the surface of the protective window and diode surface. We use two spherical mirrors ( $R=10$  mm), to retroreflect the reflected light onto the detector improving the quantum efficiency by 7%, i.e., to 95%. Quarter-wave plates are used to prevent the returning light from reaching the OPO cavity. For the local oscillator power of  $400 \mu\text{W}$  electronic noise of the detector is 14 dB below the standard quantum limit. Electronic noise was subtracted from all of the traces.

As described in the theory section, when fluctuations in frequency are included, the degree of squeezing is expected to depend on the relative delay through two paths: From laser to PBS1 through the local oscillator fiber, and from laser to PBS1 through amplifier, doubler, pump fiber and OPO. Insensitivity to these fluctuations is expected to occur at a “white light” condition of equal delays. Initial measurements were taken with the local oscillator fiber chosen to achieve this condition, as described in detail in Sec. IV.

Noise measurements were performed at fixed frequency, zero span of the spectrum analyzer. The degree of squeezing we observe matches the above-mentioned gain and loss parameters for the demodulation frequencies 3 MHz and higher. The highest level of squeezing of  $-2.5$  dB we observe at the demodulation frequency of 2 MHz shown in Fig. 2.

### III. RELATIVE LOCK QUALITY

We note that the achieved linewidth for the stabilized diode laser (400 kHz) is an order of magnitude below the linewidths of the doubling cavity (14 MHz) and OPO cavity (8 MHz). This justifies treating the frequency fluctuations of the laser as quasistationary when determining the effect on squeezing. Another treatment of phase noise has been discussed [12], but is far more involved and does not consider group delay effects. At the same time, while fast feedback to the laser current allows a high-bandwidth lock of the laser and doubling cavity, there is no corresponding fast control for locking of the OPO cavity to the laser frequency. Also, the PDH scheme, which achieves a very good signal by injecting through the cavity output coupler, cannot be used in many squeezing experiments because it would contaminate the squeezed light. For these reasons, the active stabilization of the OPO cavity may be an important factor in the performance of squeezing experiments.

### IV. THEORY

The theoretical description adapts the treatment of Collet and Gardiner [13] to model the nonlinear interaction inside the OPO cavity. Here we assume that the frequency drift of the diode laser is slow on the time scale of the decay of light inside the OPO cavity. This quasimonochromatic treatment describes a single mode laser drifting slowly within a finite linewidth. In such a system frequency fluctuations lead to a fluctuating phase shift between the squeezed mode and the local oscillator mode. Our calculation modifies [13] by including a relative detuning  $\Delta\omega$  between pump laser and OPO cavity caused by the random frequency drifts. As in [13], we start from the quantum Langevin equation of the OPO cavity

$$\dot{a} = -\frac{i}{\hbar}[a, H_{\text{sys}}] - (k_1 + k_2)a + \sqrt{2k_1}a_{v1} + \sqrt{2k_2}a_{v2}, \quad (1)$$

where  $a$  and  $a^\dagger$  denote annihilation and creation operators of the cavity mode with frequency  $\omega_0$ ,  $k_1$  and  $k_2$  denote the loss rates due to output coupler and intracavity losses, and  $a_{v1}$  and  $a_{v2}$  denote the annihilation operators of the (vacuum) field entering the cavity due to output coupler and intracavity losses. The Hamiltonian operator of the system is

$$H_{\text{sys}} = \hbar\omega_0 a^\dagger a + \frac{i\hbar}{2}[\epsilon e^{-i\omega_p t}(a^\dagger)^2 - \epsilon^* e^{i\omega_p t} a^2], \quad (2)$$

where the first term describes the energy of photons inside the cavity while the second term models the nonlinear interaction induced by the pump field with frequency  $\omega_p$ . The phase of the nonlinear coupling constant  $\epsilon = |\epsilon|e^{i\phi}$  is determined by the phase of the pump field,  $\phi$ . Furthermore, we assume that the squeezed mode is detuned from the cavity resonance by  $2\Delta\omega \equiv \omega_p - 2\omega_0$ . By performing the equivalent calculation as in [13] we finally reach the Bogoliubov transformation from input to output fields

$$\begin{aligned} \tilde{a}_{\text{out}}(\omega + \Delta\omega) &= [A_1 \tilde{a}_{v1}(\omega + \Delta\omega) + A_2 \tilde{a}_{v2}(\omega + \Delta\omega) \\ &\quad + C_1 \tilde{a}_{v1}^\dagger(-\omega + \Delta\omega) + C_2 \tilde{a}_{v2}^\dagger(-\omega + \Delta\omega)] B^{-1}, \end{aligned} \quad (3)$$

where

$$A_1 = \eta^2 - (1 - \eta - i\Omega)^2 + \Delta\Omega(2\eta i - \Delta\Omega) + |\alpha|^2, \quad (4)$$

$$A_2 = 2\sqrt{\eta(1-\eta)}[i(-\Omega + \Delta\Omega) + 1], \quad (5)$$

$$C_1 = 2\eta\alpha, \quad (6)$$

$$C_2 = 2\alpha\sqrt{\eta(1-\eta)}, \quad (7)$$

$$B = (1 - i\Omega)^2 + \Delta\Omega^2 - |\alpha|^2. \quad (8)$$

We have introduced  $\tilde{a}$  as the operators in rotating frame, and scaled all frequencies and rates to the cavity linewidth, i.e., demodulation (detection) frequency  $\Omega = \frac{\omega}{k_1+k_2}$ , detuning  $\Delta\Omega = \frac{\Delta\omega}{k_1+k_2}$ , cavity escape efficiency  $\eta = \frac{k_1}{k_1+k_2}$ ,  $1 - \eta = \frac{k_2}{k_1+k_2}$ , and pump amplitude  $\alpha = \frac{\epsilon}{k_1+k_2}$ . The squeezing spectrum  $S(\Omega)$  can be deduced from Eq. (3) using  $q_\theta = \frac{1}{\sqrt{2}}(\tilde{a}_{\text{out}}e^{-i\theta} + \tilde{a}_{\text{out}}^\dagger e^{i\theta})$ ,

$$\begin{aligned} S(\Omega) &= 1 + 2\eta_{\text{det}} \langle :q_\theta q_\theta: \rangle \\ &= 1 + \frac{8\eta_{\text{det}}\eta|\alpha|^2}{|B|^2} \left[ 1 + \frac{(1 - \Delta\Omega^2 + \Omega^2 + |\alpha|^2)}{2|\alpha|} \right. \\ &\quad \left. \times \cos(\Delta\phi + 2\Delta\omega\tau_D) + \frac{\Delta\Omega}{|\alpha|} \sin(\Delta\phi + 2\Delta\omega\tau_D) \right], \end{aligned} \quad (9)$$

where  $\theta = \theta_0 + \Delta\omega\tau_D$  denotes the phase of the local oscillator, with  $\theta_0$  being the phase of the local oscillator in the white light configuration and  $2\Delta\omega\tau_D$  being the phase shift for a detuned local oscillator when the light is delayed for  $\tau_D$  from the white light configuration. Furthermore,  $\Delta\phi = 2\theta_0 - \phi$  denotes the relative phase between the phase of the local oscillator in the white light configuration and of the pump laser of the OPO,  $\alpha = |\alpha|e^{i\phi}$ . Best squeezing is obtained for the phase that gives

$$\tan(\Delta\phi + 2\Delta\omega\tau_D) = \frac{2\Delta\Omega}{1 - \Delta\Omega^2 + \Omega^2 + |\alpha|^2} \quad (10)$$

which due to the cavity dispersion depends on the detuning of the pump laser  $\Delta\Omega$ . The right-hand side of the equation represents the delay in the OPO cavity. In first order of the detuning the squeezing phase is

$$\Delta\phi + 2\Delta\omega\tau_D = \pi + \frac{2}{1 + \Omega^2 + |\alpha|^2} \Delta\Omega. \quad (11)$$

This dispersion can be compensated by delaying the local oscillator before the homodyne detection. A delay line of length  $l$  and group index  $n_g$  will introduce the phase shift

$$2\Delta\omega\tau_D = 2(k_1 + k_2) \frac{ln_g}{c} \Delta\Omega. \quad (12)$$

Thus for delay length

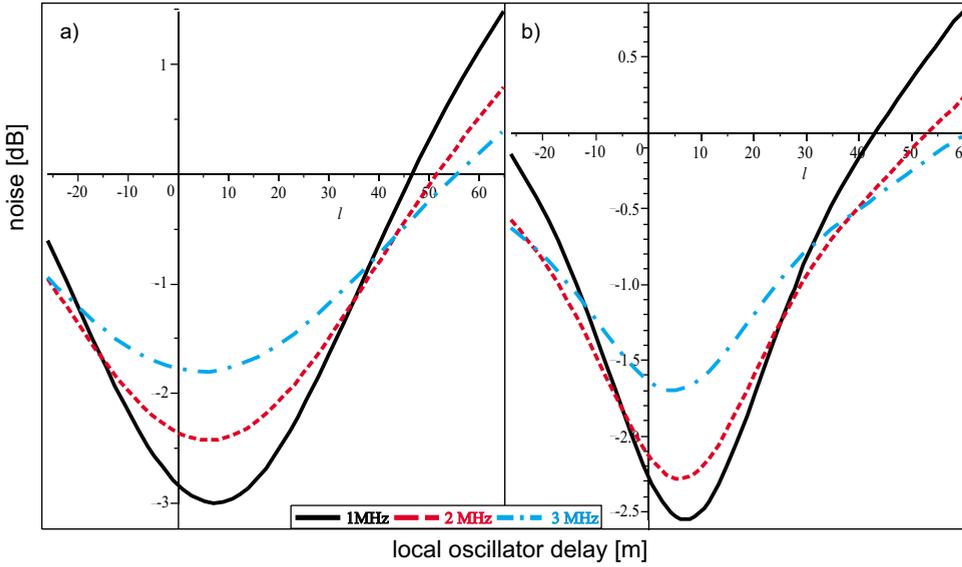


FIG. 3. (Color online) Squeezing vs delay for three different detection frequencies 1 MHz, 2 MHz, and 3 MHz depicted black (solid), red (dashed), and blue (dot-dashed), respectively; (a) models the laser spectrum as Gaussian of linewidth 700 kHz full width half-maximum, (b) models the laser spectrum as Lorentzian of linewidth 300 kHz (FWHM).

$$l = \frac{c}{n_g(k_1 + k_2)(1 + \Omega^2 + |\alpha|^2)} \quad (13)$$

the homodyne detection will be performed, to first order, at the correct squeezing phase  $\Delta\phi = \pi$  even for the detuned pump. The dispersion in the OPO cavity and therefore also the compensation length depends on the detection frequency  $\Omega$ . For higher detection frequency a shorter compensation delay is necessary. Assuming a slowly drifting laser with power spectral density of  $\rho(\Delta\Omega)d\Delta\Omega$  the obtained squeezing can be modeled by averaging the homodyne power spectrum  $S(\Omega)$  for phase  $\Delta\phi = \pi$  over  $\Delta\Omega$ . The averaged squeezing spectrum

$$\bar{S}(\Omega) = \int_{-\infty}^{+\infty} S(\Omega, \Delta\Omega)\rho(\Delta\Omega)d\Delta\Omega \quad (14)$$

is plotted in Fig. 3 for a Gaussian and a Lorentzian linewidth  $\rho(\Delta\Omega)$ . We note that physically  $\Delta\Omega$  is the mismatch between one-half of the pump frequency and the OPO cavity frequency scaled to the cavity linewidth, and thus both laser frequency fluctuations and OPO cavity fluctuations will contribute to  $\rho(\Delta\Omega)$ . The shift of optimum squeezing to positive delay is due to the existence of the delay introduced by OPO.

### V. DELAY CONSIDERATIONS

We note that in Eqs. (3) and (13),  $\tau_D$  is the group delay between the local oscillator and the pump light at the cavity. As both local oscillator and pump are ultimately derived from the same laser, we can identify  $\tau_D = 0$  as a “white-light” condition in a Mach-Zehnder-topology interferometer. The light in the squeezing path passes the tapered amplifier, doubling cavity, mode-matching fiber, lengths of free-space propagation, and the OPO cavity. The light in the local oscillator path passes lengths of free-space propagation and a mode-cleaning fiber (which we use to introduce the desired delays).

In presenting the results, the “zero” of  $\tau_D$  is taken to be when the total delay in the local oscillator path, as calculated

from measurements of fiber and free-space lengths, is equal to the combined delays in the amplifier, doubling cavity, fiber, and free space. We do not include the OPO cavity delay because this depends on  $\Omega$ , as presented in the theory and shown in Fig. 3.

The delay introduced by the doubling cavity is the cavity group delay at line center

$$\tau = \frac{1}{\pi\delta\nu}. \quad (15)$$

The measured doubling cavity linewidth is  $\delta\nu = 14$  MHz. To delay the local oscillator we have used fibers with group index  $n_g = 1.5$ .

We note that, as the laser and doubling cavity are mutually locked, it is not obvious how the doubling cavity delay should be included. While the light is obviously propagating from laser through doubling cavity, a frequency fluctuation in the doubling cavity will, via the current feedback, affect the laser frequency. We choose to include the cavity delay, in the squeezing path because it gives best agreement with the data presented below.

### VI. MEASURED SQUEEZING VERSUS DELAY

We have performed a series of measurement where a controllable delay was introduced in the path of the local oscillator with intention to (i) measure the level of squeezing in the white light configuration, (ii) see the effect of the change of delay on the level of squeezing. The results are presented in Fig. 4.

We performed the measurements of the quadrature variance for every 4 meters added in the local oscillator path starting from the proximity of the balanced delay configuration. Final fiber length was 60 m longer than the balanced configuration. Due to the limited pump power and large fiber losses for the blue light, measurements at negative delay were not feasible. We measured squeezing vs delay for three different demodulation frequencies 1 MHz, 2 MHz, and 3 MHz (Fig. 4).

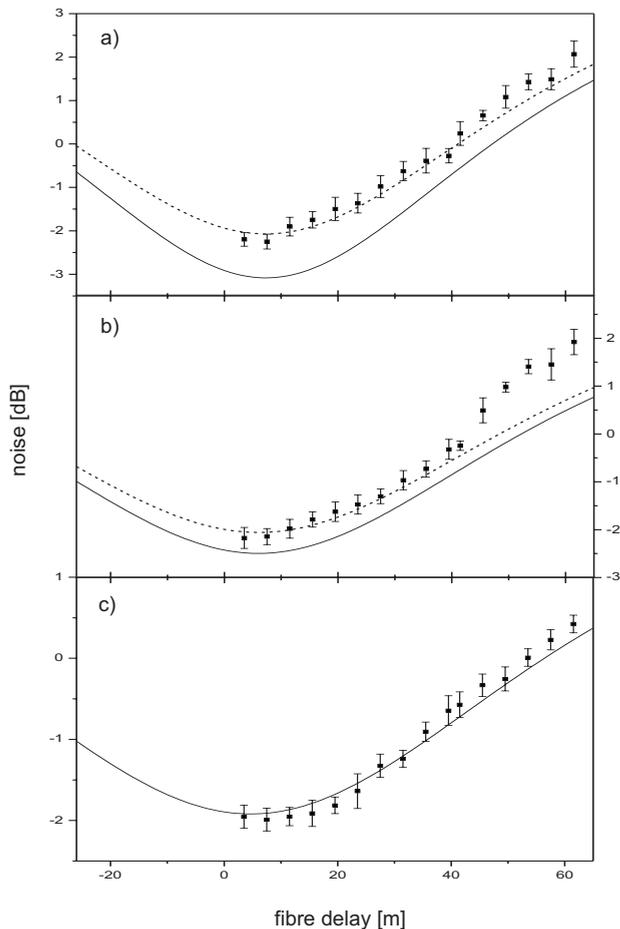


FIG. 4. Squeezing dependent on the path mismatch measured for three different detection frequencies: (a) at 1 MHz demodulation frequency, (b) at 2 MHz demodulation frequency, (c) at 3 MHz demodulation frequency. The points show the experimental data, the solid lines the predicted level of squeezing for the parameters measured in the experiments using a Gaussian profile of 700 kHz linewidth (FWHM), the dashed lines show the theoretical level of squeezing for the same parameters as the solid line with additional technical noise independent of the relative delay. The error bars represent standard deviation over series of identical measurements.

The experimental results show minima at positive delay as predicted by theory. Equation (13) predicts  $l = \{7.3, 6.0, 4.7\}$  m shift for demodulation frequencies  $\{1, 2, 3\}$  MHz, respectively. Here we assume that the doubling cavity delay is equal to the delay which the cavity introduces at the resonance. Naturally this delay does not depend on the demodulation frequency.

The theoretical curves in Fig. 4 are obtained using all experimental parameters as stated above, but varying the width of  $\rho(\Delta\Omega)$  as the only free parameter. Of two different profiles treated in the theory the comparison with the Gaussian reflects the shape of the experimental curve more closely than the Lorentzian profile. We see good agreement, especially at 3 MHz demodulation frequency, for a Gaussian spectrum of 700 kHz (FWHM). Using the in-loop signal from the laser lock to a saturated-absorption reference, we find a 400 kHz laser linewidth. A similar measurement of the distribution of  $\omega_0 - \omega_{\text{laser}}$  can be made using the OPO cavity

locking signal. Under the conditions of the squeezing measurements, however, the locking signal was too weak to extract a meaningful signal, largely because we cannot inject through the output mirror as in the PDH technique. We can place a lower limit of 300 kHz on the width of  $\rho(\Delta\Omega)$  based on PDH locking of the same cavity, and the 700 kHz estimate for the width of  $\rho(\Delta\Omega)$  appears reasonable.

On the other hand, the level of squeezing we observe in the 1 MHz and 2 MHz measurements is smaller than predicted by theory. This might be caused by the light back-reflected from the end faces of the nonlinear crystal contaminating the squeezed light. If we assume that this noise is independent of the relative delay, it can be modeled by a constant offset to our theoretical squeezing curves. With an offset of  $(+0.07, +0.03)$  relative to the standard quantum limit for (1, 2) MHz, respectively, the theory for a Gaussian laser spectrum of 700 kHz fits well in shape and amplitude to our measured data as shown in Fig. 4. By solving the problem of noise which causes the decrease of squeezing in the 1 MHz and 2 MHz measurements one could in agreement with the theory detect more than 5 dB of noise reduction at the OPO output.

## VII. CONCLUSION

We have demonstrated quadrature and polarization squeezing using a subthreshold OPO and a frequency-doubled diode laser for a pump. We have investigated and optimized the squeezing properties by using a delayed local oscillator. We adapted the theoretical description of Collet and Gardiner [13] under the assumption of slow frequency fluctuations. The theoretical description can be used to model random frequency fluctuations of the laser but also the problem of optimization of the OPO cavity stabilization. This approach showed that the OPO cavity exhibits dispersive behavior which causes a delay of the squeezed light. Optimum squeezing is observed if the squeezed light is in a white light configuration with respect to the local oscillator. Experimental results confirmed that the vacuum mode of the OPO is also taking part in the delay line. This investigation shows that, by taking into account the balancing and the delay lines, diode laser sources can be used for producing quadrature and polarization squeezing in an OPO. Since diode lasers are much cheaper and simpler to operate our work brings portable inexpensive squeezing devices for application in e.g., precision measurements into reach.

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