

## Quantum Polarization Spectroscopy of Ultracold Spinor Gases

K. Eckert,<sup>1</sup> Ł. Zawitkowski,<sup>2</sup> A. Sanpera,<sup>3</sup> M. Lewenstein,<sup>4</sup> and E. S. Polzik<sup>5,6</sup>

<sup>1</sup>*Grup de Física Teòrica, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*

<sup>2</sup>*Centrum Fizyki Teoretycznej, Polska Akademia Nauk, Warszawa 02668, Poland*

<sup>3</sup>*ICREA and Grup de Física Teòrica, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Spain*

<sup>4</sup>*ICREA and ICFO-Institut de Ciències Fotòniques, E-08860 Castelldefels, Barcelona, Spain*

<sup>5</sup>*ICFO-Institut de Ciències Fotòniques, E-08860 Castelldefels, Barcelona, Spain*

<sup>6</sup>*Niels Bohr Institute, Danish Quantum Optics Center–QUANTOP, Copenhagen University, Blegdamsvej 17, Copenhagen 2100, Denmark*

(Received 14 August 2006; published 9 March 2007)

We propose a method for the detection of ground state quantum phases of spinor gases through a series of two quantum nondemolition measurements performed by sending off-resonant, polarized light pulses through the gas. Signatures of various mean-field as well as strongly correlated phases of  $F = 1$  and  $F = 2$  spinor gases obtained by detecting quantum fluctuations and mean values of polarization of transmitted light are identified.

DOI: [10.1103/PhysRevLett.98.100404](https://doi.org/10.1103/PhysRevLett.98.100404)

PACS numbers: 03.75.Mn, 03.75.Hh, 03.75.Lm, 32.80.Qk

It was demonstrated several years ago that fundamental quantum spin noise of a collection of cold atoms can be measured via quantum noise limited polarization spectroscopy [1]. Since then, a quantum interface of light with optically thick atomic spin ensembles has become a promising and powerful method for a transfer of quantum information between atomic internal degrees of freedom and light. The basic concept underlying such atom-light interfaces is provided by the off-resonant coupling of the collective atomic spin, i.e., of several magnetic sublevels, to the polarization of light. In particular, such an off-resonant interaction, followed by a quantum measurement on light, has been shown to be a powerful quantum nondemolition (QND) tool to generate spin squeezed and entangled atomic states [2–6], to teleport quantum states between ensembles [7], or to propose [8,9] and realize [6] high fidelity quantum memories for light.

In the present Letter we propose to apply quantum polarization spectroscopy techniques for the detection of various quantum phases of degenerate atomic gases, i.e., with atoms having spin degrees of freedom. Such ultracold spinor gases have recently brought a new perspective to the study of magnetic systems. Seminal experiments of the MIT group with an optically trapped spin  $F = 1$  sodium condensate [10], and theory papers of Ho [11], and Ohmi and Machida [12] have triggered the use of cold atoms to study magnetic ordering and domains. Spin interaction effects are very much enhanced in the strongly correlated regime, which nowadays is reachable experimentally [13] by loading an ultracold spinor gas into an optical lattice so that the kinetic energy (tunneling) becomes small in comparison with atom-atom interactions. Then the atoms can be well described by a generalized spinor Bose-Hubbard Hamiltonian (BHH) [14–16]. In the limit of small occupation number it reproduces accurately (within the experimentally achievable regime) some of the most paradigmatic spin chain models. Experimental observation of the

rich variety of magnetic ordering present in these systems remains, however, elusive due to similar values of the scattering lengths on the different spin collision channels.

A way to determine properties of a quantum spinor gas could be to use a strong QND measurement. As shown here, a series of QND measurements using polarization spectroscopy on light transmitted through a condensate yields mean values and variances of the atomic total spin operators, thus allowing unambiguous distinction of various atomic quantum phases.

*Formalism*—We consider a sample of neutral atoms in a  $(2F + 1)$ -dimensional ground state manifold  $|F, m\rangle$ , interacting off resonantly with linearly polarized light propagating along the  $z$  direction (cf. [17]). After adiabatically eliminating excited atomic states, the interaction can be described via an effective Hamiltonian

$$\hat{H}_{\text{int}}^{\text{eff}} = - \int_0^L dz \rho A (a_0 \hat{\phi} + a_1 \hat{s}_z \hat{j}_z + a_2 [\hat{\phi}_{j_z}^2 - \hat{s}_{j_+}^2 - \hat{s}_{j_-}^2]), \quad (1)$$

where  $L$  is the length of the atomic sample. The conditions under which decoherence due to absorption of light can be neglected, such that this Hamiltonian is valid, will be discussed below. In Eq. (1),  $a_i \propto \hbar \gamma \lambda^2 c / (16 \pi A \Delta)$ , where  $A$  is the cross section of the atomic sample overlapping with the probe light,  $\Delta$  is the detuning,  $\rho$  is the (in general  $z$  dependent) atomic density,  $\lambda$  is the wavelength, and  $\gamma$  is the excited state line width.  $\hat{s}_\alpha \equiv \hat{s}_\alpha(z, t)$  ( $\hat{s}_\pm = \hat{s}_x \pm i \hat{s}_y$ ) are the components of the Stokes vector characterizing the polarization of the light pulse,  $\hat{\phi}(z, t)$  is the photonic density, and  $\hat{j} \equiv \hat{j}(z, t)$  are atomic spin operators. The term proportional to  $a_0$  corresponds to the ac Stark shift, while  $a_2 \rightarrow 0$  for values of  $\Delta$  large compared to the excited state hyperfine structure [17]. Here we assume  $a_2 = 0$  and restrict to the linear coupling between the Stokes operator and the atomic spin, which represents a QND Hamiltonian.

Using Heisenberg equations of motion, we find that  $\hat{j}_z$  is conserved. For a pulsed probe with a duration of  $\mu\text{s}$  we can thus assume  $\hat{j}_z(z, t) \equiv \hat{j}_z(z)$ , as spin diffusion happens on a much larger time scale of ms. For a probe strongly polarized along the  $x$  direction, the macroscopic Stokes operator  $\hat{S}_x$  ( $\hat{S}_\alpha = \int dt \hat{s}_\alpha$ ) can be replaced by a  $c$  number  $\hat{S}_x \approx \langle \hat{S}_x \rangle = N_P/2$ , being  $N_P$  the number of photons. Neglecting retardation effects, the propagation equation for  $\hat{S}_y$  reads  $\partial_z \hat{S}_y(z) = -a_1 A \rho \langle \hat{S}_x \rangle \hat{j}_z(z)$ . It is convenient to introduce the collective spin in  $z$  direction  $\hat{J}_z = \int_0^L dz A \rho \hat{j}_z(z)$  and to define quadrature operators via  $\hat{X}_S = \sqrt{2/N_P} \hat{S}_y$ ,  $\hat{Y}_S = \sqrt{2/N_P} \hat{S}_z$ , such that  $[\hat{X}_S, \hat{Y}_S] \approx i$ . Integrating the propagation equation gives

$$\hat{X}_S^{\text{out}} = \hat{X}_S^{\text{in}} - \frac{\kappa}{\sqrt{FN_A}} \hat{J}_z, \quad (2)$$

with  $\kappa^2 = a_1^2 N_P N_A \frac{F}{2}$  (number of atoms  $N_A = \int dz \rho A$ ). Equation (2) is valid provided that  $2\kappa^2 FN_A/N_P \ll 1$ . Fluctuations of the  $\hat{X}_S$  quadrature are calculated as

$$(\Delta \hat{X}_S^{\text{out}})^2 = \frac{1}{2} + \frac{\kappa^2}{FN_A} \int_0^L dz \int_0^L dz' \rho^2 A^2 [\langle \hat{j}_z(z) \hat{j}_z(z') \rangle - \langle \hat{j}_z(z) \rangle \langle \hat{j}_z(z') \rangle], \quad (3)$$

where  $\frac{1}{2}$  stems from the quantum fluctuations of the coherent input state of light (preparing a squeezed input state would allow to reduce this contribution).

Equations (1)–(3) allow to estimate the feasibility of the proposed methods for measurements of the fluctuations of  $\hat{J}_z$  of a spinor condensate. A strong QND measurement repeated on samples prepared in the same state yields a complete knowledge about the operator  $\hat{J}_z$ . The strength of the interaction is determined by the constant  $\kappa$ . To obtain useful information about the spin fluctuations, the second term in Eq. (3) must be large compared to  $\frac{1}{2}$ , the quantum noise of a coherent probe. For the sake of this estimate we take a ferromagnetic spinor condensate (also referred to as a coherent spin state outside the context of quantum gases), for which the second term is equal to  $\kappa^2/2$  [3,4]. The interaction constant can be expressed [18] as  $\kappa^2 = \alpha \eta$ , where  $\eta$  is the probability of spontaneous excitation caused by the off-resonant probe and  $\alpha$  is the resonant optical depth of the sample.  $\eta$  describes the decoherence which must be kept small by choosing the detuning and/or the strength of the probe pulse, in order to minimize the distortion of  $\hat{J}_z$  and to validate the use of Hamiltonian Eq. (1). The optimal value for a QND measurement of  $\hat{J}_z$  is  $\kappa^2 = \sqrt{\alpha}/2$  [18]. Hence for an optically thick spinor condensate with  $\alpha \gg 1$  the proposed method works well. For degenerate gases, typical values of  $\alpha = 300$  or even more have been demonstrated [19].

The most general Hamiltonian describing a spinor atomic sample is given by

$$\hat{H}_{\text{at}} = \int d\mathbf{r} \sum_{m=-F}^F \hat{\Psi}_m^\dagger(\mathbf{r}) \left( -\frac{\hbar^2 \Delta^2}{2M} + U_{\text{trap}}(\mathbf{r}) \right) \hat{\Psi}_m(\mathbf{r}) + \hat{V}_{\text{int}}, \quad (4)$$

where the field operator  $\hat{\Psi}_m^\dagger(\mathbf{r})$  creates a particle with spin projection  $m$  at position  $\mathbf{r}$  and  $\hat{V}_{\text{int}}$  describes the atom-atom interactions. For two identical spin- $F$  bosons interacting via  $s$ -wave collisions,  $\hat{V}_{\text{int}} = \sum_{M=0}^F g_{2M} \hat{P}_{2M}$ , with  $\hat{P}_{2M}$  being the projector onto the subspace with total spin  $F^{\text{tot}} = 2M$ , and  $g_{2M}$  the interaction strength for the given spin channel. For a spatially uniform trapping potential, condensation occurs in the zero momentum state, and variational ground states are obtained from a trial wave function [20]

$$|\xi\rangle = \frac{1}{\sqrt{N_A!}} \left[ \sum_{m=-F}^F \xi_m \hat{a}_{\mathbf{k}=0,m}^\dagger \right]^{N_A} |\text{vac}\rangle, \quad (5)$$

with the complex components of the vector  $\xi$  as parameters.  $\hat{a}_{\mathbf{k}=0,m}^\dagger$  creates a particle with spin projection  $m$  in the  $\mathbf{k} = \mathbf{0}$  state. Ground states have been discussed in detail for  $F = 1$  [11],  $F = 2$  [20], and  $F = 3$  [21]. Given the spinor  $\xi$ , means and variances are calculated as

$$\begin{aligned} \langle \hat{X}_S^{\text{out}} \rangle &= -\frac{\kappa \sqrt{N_A}}{\sqrt{F}} \sum_{m=-F}^F m |\xi_m|^2, \\ (\Delta \hat{X}_S^{\text{out}})^2 &= \frac{1}{2} + \frac{\kappa^2}{F} \left[ \sum_{m=-F}^F m^2 |\xi_m|^2 - \left( \sum_{m=-F}^F m |\xi_m|^2 \right)^2 \right]. \end{aligned} \quad (6)$$

In the strongly correlated regime, Eq. (4) reduces to a spinor BHH. For an integer filling factor and considering tunneling perturbatively up to second order, an effective Hamiltonian with nearest neighbor spin-spin interaction arises. For such a situation, ground state configurations have been analyzed in [14] for  $F = 1$  and in [16,22] for  $F = 2$ . Equation (3) reduces then to a sum over correlations between all pairs of atoms:

$$(\Delta \hat{X}_S^{\text{out}})^2 = \frac{1}{2} + \frac{\kappa^2}{FN_A} \sum_{k,l=1}^{N_A} [\langle \hat{j}_z^k \hat{j}_z^l \rangle - \langle \hat{j}_z^k \rangle \langle \hat{j}_z^l \rangle]. \quad (7)$$

Notice that in  $\hat{H}_{\text{int}}^{\text{eff}}$ , Eq. (1), light couples to individual atoms. Thus for more than one atom per site,  $\hat{j}^k$  denotes the spin operators of the  $k$ th atom, not the total spin at site  $k$ .

The procedure described so far allows to obtain mean and variance of  $\hat{J}_z$ . Reading an orthogonal component of the atomic spin vector allows for a better discrimination of distinct phases. Since the interaction with the first light pulse modifies the atomic spin state, here we assume to have at our disposal a second, identically prepared atomic sample. A light pulse incident in the  $y$  direction, again strongly  $x$  polarized, corresponds to exchange  $\hat{j}_z \rightarrow -\hat{j}_y$ ,  $\hat{j}_y \rightarrow -\hat{j}_z$  in Eqs. (1)–(3). Thus it allows to read out the  $y$

component of the atomic spin vector, with the corresponding quadrature denoted as  $\hat{X}_U$ .

*Detecting spin-1 quantum phases.*—In the mean-field case, a gas of  $F = 1$  atoms has two possible ground states [11]: a ferromagnetic one with all spins having maximal spin projection in some direction  $(\cos\alpha \sin\beta, \sin\alpha \sin\beta, \sin\beta \sin\beta)$ , and thus  $\langle \hat{j} \rangle \neq \mathbf{0}$ , and a polar one with  $\langle \hat{j} \rangle = \mathbf{0}$ . Inserting the ferromagnetic spinor  $\xi_f = (e^{-i\alpha} \cos^2 \frac{\beta}{2}, \sqrt{2} \cos \frac{\beta}{2} \sin \frac{\beta}{2}, e^{i\alpha} \sin^2 \frac{\beta}{2})$  into Eqs. (6) leads to  $\langle \hat{X}_S^{\text{out}} \rangle = -\kappa \sqrt{N_A} \cos\beta$ ,  $\langle \hat{X}_U^{\text{out}} \rangle = \kappa \sqrt{N_A} \sin\alpha \sin\beta$ ,  $(\Delta \hat{X}_S^{\text{out}})^2 = (1 + \kappa^2 \sin^2 \beta)/2$ ,  $(\Delta \hat{X}_U^{\text{out}})^2 = (1 + \kappa^2 [1 - \sin^2 \beta \sin^2 \alpha])/2$ . These equations are valid provided the mean polarization of the probe light remains to be  $x$  polarized which is true under the feasible assumption  $N_A \ll N_p$ . For the polar phase, the spinor can be parameterized as  $\xi_p = (-e^{-i\alpha} \sin\beta, \sqrt{2} \cos\beta, e^{i\alpha} \sin\beta)/\sqrt{2}$ , and we have  $\langle \hat{X}_S^{\text{out}} \rangle = 0 = \langle \hat{X}_U^{\text{out}} \rangle$  and  $(\Delta \hat{X}_S^{\text{out}})^2 = (1 + 2\kappa^2 \sin^2 \beta)/2$ ,  $(\Delta \hat{X}_U^{\text{out}})^2 = (1 + 2\kappa^2 [1 - \sin^2 \alpha \sin^2 \beta])/2$ . Characterizing the atomic phases by the additional noise  $(\Delta \hat{X}_{S/U}^{\text{out}})^2 - \frac{1}{2} \equiv \epsilon_{S/U}^2$  imprinted on the  $\hat{X}$  quadratures, we obtain for the ferromagnetic phase  $\epsilon_U^2 = \frac{1}{2} \kappa^2 - \epsilon_S^2 \sin^2 \alpha$ , with  $0 \leq \epsilon_S^2 \leq \frac{1}{2} \kappa^2$ , and for the polar phase  $\epsilon_U^2 = \kappa^2 - \epsilon_S^2 \sin^2 \alpha$ , with  $0 \leq \epsilon_S^2 \leq \kappa^2$ . Possible values of the additional noise lie in nonoverlapping triangles in the  $(\epsilon_S^2, \epsilon_U^2)$  plane, see Fig. 1(a). Thus both phases can be distinguished through the noise imprinted on the light. In this particular case, they can also be distinguished by comparing the mean values of the light quadratures [for  $(\alpha, \beta) \neq (0, \pi/2)$ ].

For the  $F = 1$  lattice gas with a single particle per site, the effective Hamiltonian is  $\hat{H}_{\text{lat}} = \sum_{\langle kl \rangle} [\cos\delta \hat{j}^k \hat{j}^l + \sin\delta (\hat{j}^k \hat{j}^l)^2]$ , where the sum runs over nearest neighbors. We consider only the ground states of  $^{23}\text{Na}$  discussed by Imambekov *et al.* [14] (for numerical investigations see [23]): (i) for a fully polarized state the properties of the outgoing light are as in the ferromagnetic case discussed before; (ii) a state mixing total spin  $F^{\text{tot}} = 0$  and  $F^{\text{tot}} = 2$  on each bond, constructed as  $\prod_m |\xi_p\rangle_m$ , gives results as for the polar mean-field state; (iii) singlets (dimers) can be put on every second bond (in 1D), in this way breaking trans-

lational symmetry. The reduced on-site density matrix is  $\rho_k = \frac{1}{3}$ , and thus  $\langle X_S^{\text{out}} \rangle = 0$ . As  $\langle \hat{j}_k^z \hat{j}_l^z \rangle = \frac{2}{3}$  for  $k = l$ ,  $-\frac{2}{3}$  for nearest-neighbors in a singlet state, and 0 otherwise, for an even number of sites that the fluctuations are unchanged:  $(\Delta \hat{X}_S^{\text{out}})^2 = \frac{1}{2}$ . For an odd number of sites in 1D, or randomly oriented dimers in 2D,  $n$  atoms will be unpaired and thus  $(\Delta \hat{X}_S^{\text{out}})^2 = \frac{1}{2} + 2n\kappa^2/(3N_A)$ . Because of the rotational symmetry of the singlet, the same result is obtained for the  $\hat{X}_U$  quadrature. (iv) In 1D and for  $\delta = -\arctan(\frac{1}{3})$ , the ground state is a valence bond solid (VBS) state [24]. Its two-site correlations decay as  $\langle \hat{j}_z^k \hat{j}_z^l \rangle = \frac{4}{3} \times (-\frac{1}{3})^{|k-l|}$  ( $k \neq l$ ) [24]. Since the VBS is nonmagnetized, the means of the quadrature components remain unchanged. As the sum of  $\langle \hat{j}_z^k \hat{j}_z^l \rangle$  over all pairs of atoms gives  $\frac{2}{3}$  independent of the number of sites, there is no detectable change in the fluctuations:  $(\Delta \hat{X}_S^{\text{out}})^2 = \frac{1}{2} + 2\kappa^2/3N_A$ . Thus distinguishing between a dimer and a VBS state is difficult with this method, but in principle possible for small  $N_A$ .

For two atoms per lattice site, in the limit of vanishing tunneling the ground state consists of noninteracting singlets on each site. As tunneling is increased, on-site states with total spin 2 become important. Using a variational ansatz  $\prod_k |\psi\rangle_k$  with  $|\psi\rangle_k = \cos\Theta |F^{\text{tot}} = 0, F_z^{\text{tot}} = 0\rangle + \sin\Theta |F^{\text{tot}} = 2, F_z^{\text{tot}} = 0\rangle$ , a sharp jump of  $\sin\Theta$  from 0 to a nonzero value is found as tunneling is increased [14]. Evaluating the quadrature operators of the outgoing light, we find  $\langle \hat{X}_S^{\text{out}} \rangle = 0 = \langle \hat{X}_U^{\text{out}} \rangle$ , but modified noise properties:  $(\Delta \hat{X}_S^{\text{out}})^2 = (1 + 3\kappa^2 \sin^2 \Theta \sin^2 \beta)/2$ ,  $(\Delta \hat{X}_U^{\text{out}})^2 = (1 + 3\kappa^2 \sin^2 \Theta [1 - \sin^2 \alpha \sin^2 \beta])/2$ , where  $\alpha, \beta$  parameterize the direction of the component with  $F^{\text{tot}} = 2$ . The sharp change in the nature of the ground state manifests clearly in the noise properties of the outgoing light. Let us emphasize that as light couples to single atoms, here fluctuations are different from those arising from fully polarized  $F = 2$  atoms, as will be discussed now.

*Detecting spin-2 quantum phases.*—For  $F = 2$ , there are three different ground state phases in the mean-field case [20]: (i) ferromagnetic,  $\langle \hat{j} \rangle \neq \mathbf{0}$ , characterized by spin projection  $m = 2$  (a) or  $m = 1$  (b) in some direction, (ii) polar, characterized by  $\sigma = \sum_m (-1)^m \xi_m \xi_{-m} \neq 0$

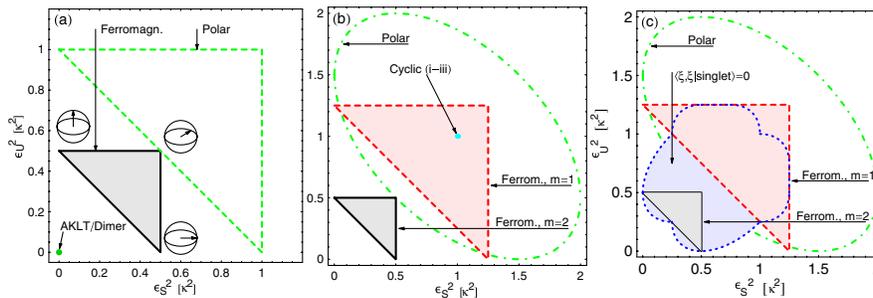


FIG. 1 (color online). Possible combinations of additional fluctuations  $\epsilon_S^2$  and  $\epsilon_U^2$  imprinted on the light for the ground state phases of the  $F = 1$  spinor gas in a uniform trap and in an optical lattice (a) and for  $F = 2$  atoms in a uniform trap (b) and in an optical lattice (c). Filled areas denote cases where the mean of  $\langle X_S^{\text{out}} \rangle$  and/or  $\langle X_U^{\text{out}} \rangle$  is (generically) nonzero. The spheres in (a) illustrate the directions of the spinor for the extremal points of the ferromagnetic phase.

TABLE I. Spinors  $\xi$  for the mean-field ground state phases in the spin-2 case [20] and corresponding results for means  $\langle \hat{X}_{S/U}^{\text{out}} \rangle$  and additional fluctuations  $\epsilon_{S/U}$ . Abbreviations  $s_x = \sin \frac{x}{2}$ ,  $c_x = \cos \frac{x}{2}$ ,  $S_x = \sin x$ ,  $C_x = \cos x$  are used.

(ia) Ferromagnetic $m = 2$ — $\xi = (e^{-i2\alpha} s_\beta^4, 2e^{-i\alpha} s_\beta^3 c_\beta, \sqrt{\frac{3}{4}} s_\beta^2, 2e^{i\alpha} s_\beta c_\beta^2, e^{i\alpha} c_\beta^4)$ $\langle \hat{X}_S^{\text{out}} \rangle = -\sqrt{2N} C_\beta$ , $\langle \hat{X}_U^{\text{out}} \rangle = -\sqrt{2N} S_\alpha S_\beta$ , $\epsilon_S^2 = \frac{1}{2} \kappa^2 S_\beta^2$ , $\epsilon_U^2 = \frac{1}{2} \kappa^2 (1 - S_\alpha^2 S_\beta^2)$
(ib) Ferromagnetic $m = 1$ — $\xi = (2e^{-i\alpha} s_\beta^3 c_\beta, -e^{-i\alpha} s_\beta^2 (1 + 2C_\beta), -\sqrt{\frac{3}{4}} S_\beta C_\beta, -e^{-\alpha} c_\beta^2 (-1 + 2C_\beta), 2e^{2i\alpha} s_\beta c_\beta^3)$ $\langle \hat{X}_S^{\text{out}} \rangle = -\frac{\sqrt{N}}{2} C$ , $\langle \hat{X}_U^{\text{out}} \rangle = -\frac{\sqrt{N}}{2} S_\alpha S_\beta$ , $\epsilon_S^2 = \frac{5}{4} \kappa^2 S_\beta^2$ , $\epsilon_U^2 = \frac{5}{4} \kappa^2 (1 - S_\alpha^2 S_\beta^2)$
(ii) Polar— $\xi = (e^{i\phi} S_\theta S_\psi, e^{i\chi} S_\theta C_\psi, \sqrt{2} C_\theta, -e^{-i\chi} S_\theta C_\psi, e^{-i\phi} S_\theta S_\psi)$ $\langle \hat{X}_S^{\text{out}} \rangle = 0$ , $\langle \hat{X}_U^{\text{out}} \rangle = 0$ , $\epsilon_S^2 = \frac{1}{2} \kappa^2 S_\theta^2 (1 + 3S_\psi^2)$ , $\epsilon_U^2 = \frac{1}{2} \kappa^2 (3 - 2S_\psi^2 + 3C_\chi^2 C_\psi^2 C_\theta^2 - \sqrt{3} C_\phi S_{2\theta} S_\psi)$
(iii) Cyclic—as [20], Eq. (60–62), for zero magnetic field: $\langle \hat{X}_S^{\text{out}} \rangle = 0$ , $\langle \hat{X}_U^{\text{out}} \rangle = 0$ , $\epsilon_S^2 = \kappa^2$ , $\epsilon_U^2 = \kappa^2$

and,  $\langle \hat{j} \rangle = 0$ , and (iii) cyclic, having  $\sigma = 0$  and  $\langle \hat{j} \rangle = 0$ . Our results are summarized in Table I and Fig. 1(b). Ferromagnetic and polar phases can be perfectly distinguished while the cyclic phase lies within the polar one.

Finally, for the  $F = 2$  lattice gas with a single particle per site, we follow the discussion in [16]. The effective Hamiltonian contains up to the fourth power of the Heisenberg interaction:  $\hat{H}_{\text{lat}} = \sum_{\langle kl \rangle} \sum_{\alpha=1}^4 \lambda_\alpha (\hat{j}^k \hat{j}^l)^\alpha$ . We will discuss here only some of the possible ground state phases arising for various combinations of  $\lambda_\alpha$ : (i) for products of either ferromagnetic or polar on-site states results are as in the mean field case; (ii) an antiferromagnetic state is formed as  $|\xi_m, \xi_{-m}, \xi_m, \dots\rangle$ , with  $|\xi_m\rangle$  ( $|\xi_{-m}\rangle$ ) being the on-site state with spin projection  $m$  ( $-m$ ) in some direction. As such Néel states are nonmagnetized, the means of the outgoing  $\hat{X}$  quadratures remain unchanged, but fluctuations are as for ferromagnetic states. Néel states can thus be distinguished from ferromagnetic ones due to different means, but cannot always be distinguished from polar states; (iii) states fulfilling  $\langle \text{singlet} | \xi, \xi \rangle = 0$ , corresponding to  $\sigma = 0$ . The cyclic states discussed in the mean-field case are particular instances of this phase, which, however, is larger because the spin projection is not restricted to zero. Possible combinations of fluctuations for the different phases are summarized in Fig. 1(c).

*Conclusions.*—We have presented a method to detect different magnetic orders arising in ultracold spinor gases. It is based on an atom-light interface, realized through the off-resonant interaction of a strongly polarized pulse of light and an ultracold ensemble. Fluctuations imprinted on the quadratures of the outgoing light contain information on the total atomic spin components. By comparing the fluctuations arising when the light impinges on the atomic sample from different directions, most ground state phases can be distinguished. We plan to apply our analysis to the complete classification of the mean-field states for arbitrary  $F$  presented recently by Barnett *et al.* [22].

We thank K. Bongs, E. Demler, A. Polls, and K. Sengstock for discussions. We acknowledge support from DPG (Nos. SFB 407, SPP 1116), EU Programmes “SCALA,” “COVAQIAL,” “QAP,” ESF Network QUDEDIS, and Spanish MEC contracts FIS 2005-04627,

-01369, and EX 2005-0830.

- [1] J. L. Sørensen, J. Hald, and E. S. Polzik, Phys. Rev. Lett. **80**, 3487 (1998).
- [2] A. Kuzmich, N. P. Bigelow, and L. Mandel, Europhys. Lett. **42**, 481 (1998).
- [3] A. Kuzmich, L. Mandel, and N. P. Bigelow, Phys. Rev. Lett. **85**, 1594 (2000).
- [4] J. M. Geremia, J. K. Stockton, and H. Mabuchi, Science **304**, 270 (2004).
- [5] B. Julsgaard, A. Kozhokin, and E. S. Polzik, Nature (London) **413**, 400 (2001).
- [6] B. Julsgaard *et al.*, Nature (London) **432**, 482 (2004).
- [7] L.-M. Duan *et al.*, Phys. Rev. Lett. **85**, 5643 (2000).
- [8] A. E. Kozhokin, K. Mølmer, and E. Polzik, Phys. Rev. A **62**, 033809 (2000).
- [9] M. Fleischhauer and M. D. Lukin, Phys. Rev. A **65**, 022314 (2002).
- [10] D. M. Stamper-Kurn *et al.*, Phys. Rev. Lett. **80**, 2027 (1998).
- [11] T.-L. Ho, Phys. Rev. Lett. **81**, 742 (1998).
- [12] K. Machida and T. Ohmi, J. Phys. Soc. Jpn. **67**, 1822 (1998).
- [13] O. Mandel *et al.*, Phys. Rev. Lett. **91**, 010407 (2003).
- [14] A. Imambekov, M. Lukin, and E. Demler, Phys. Rev. A **68**, 063602 (2003).
- [15] S. K. Yip, Phys. Rev. Lett. **90**, 250402 (2003).
- [16] Ł. Zawitkowski *et al.*, cond-mat/0603273.
- [17] B. Julsgaard, Ph.D. thesis, University of Aarhus, Denmark, 2003; D. V. Kupriyanov *et al.*, Phys. Rev. A **71**, 032348 (2005).
- [18] K. Hammerer *et al.*, Phys. Rev. A **70**, 044304 (2004).
- [19] W. Ketterle, D. S. Durfee, and D. M. Stamper-Kurn, in *Bose-Einstein Condensation in Atomic Gases*, Proceedings of the International School of Physics “Enrico Fermi,” Course CXL, edited by S. Stringari, M. Inguscio, and C. E. Wieman (CIOS Press, Amsterdam 1999).
- [20] M. Ueda and M. Koashi, Phys. Rev. A **65**, 063602 (2002).
- [21] R. B. Diener and T.-L. Ho, Phys. Rev. Lett. **96**, 190405 (2006).
- [22] R. Barnett, A. Turner, and E. Demler, Phys. Rev. Lett. **97**, 180412 (2006).
- [23] M. Rizzi *et al.*, Phys. Rev. Lett. **95**, 240404 (2005).
- [24] I. Affleck *et al.*, Phys. Rev. Lett. **59**, 799 (1987).